# Nondiffractive propagation of light in photonic crystals 

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#### Abstract

We investigate nondiffractive propagation of electromagnetic radiation, including visible light, through materials with a periodic space modulation of the refraction index, i.e., through photonic crystals. We calculate analytically and numerically the regimes where the dominating order of diffraction vanishes, i.e., the light beams of arbitrary width propagate without diffractive broadening and, equivalently, arbitrary light patterns can propagate without diffractive "smearing." We investigate the subdiffractive light propagation, where the propagation is governed by the higher (fourth) diffraction order, when the dominating order of diffraction vanishes.


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## I. INTRODUCTION

Photonic crystals, i.e., the materials with refraction index periodically modulated in space on a wavelength scale [1], are well-known to modify the properties of light propagation. Photonic crystals can introduce photonic band gaps in radiation spectra, i.e., they can serve as light conductors or insulators [2]. Photonic crystals can modify the dispersion of the light, i.e., they can significantly reduce its group velocity [3]. We concentrate in this paper on the other, less known property of the photonic crystals that the diffraction of the light can alter substantially, and, in particular, can vanish in propagation through particularly prepared photonic crystals.

Diffractive broadening of light beams (and of wave envelopes in general) is a fundamental phenomenon limiting the performance of many linear and nonlinear optical devices. Geometrical interpretation of wave diffraction is as follows: light beams of arbitrary shape can be Fourier decomposed into plane waves, which in propagation acquire phase shifts depending on their propagation angles. This dephasing of the plane wave components results in a diffractive broadening of the light beam. Figure 1(a) illustrates normal diffraction in propagation through homogeneous material, where the longitudinal component of the wave vector depends trivially on the propagation angle: $k_{\|}=k_{z}=\sqrt{|\mathbf{k}|^{2}-\left|\mathbf{k}_{\perp}\right|^{2}}$, where $\mathbf{k}_{\perp}$ $=\left(k_{x}, k_{y}\right)$. In general the normal, or positive diffraction, means that the surfaces of constant frequency are concave in the wave-vector domain $\mathbf{k}=\left(k_{x}, k_{y}, k_{z}\right)$, as illustrated in Fig. 1(a).

It is known that diffraction can become negative for materials with refractive index modulated in a direction perpendicular to the propagation direction. The negative diffraction, as illustrated in Fig. 1(b), geometrically means that the surfaces of constant frequency are convex in wave-vector domain. The negative diffraction was predicted for electromagnetic radiation [4], for acoustic waves [5], and for matter waves [6] in propagation through media modulated in one spatial dimension. The change of sign of the diffraction is extremely interesting for propagation in nonlinear materials, as resulting in a change between nonlinear self-focusing and defocusing, and allowing, e.g., the bright solitons in defocusing media [7].

The disappearance of diffraction in one-dimensional periodic material has been also reported in arrays of the waveguides [8] and in the resonators with periodic modulation of refractive index in one transverse direction [9]. The disappearance of diffraction has a plausible explanation: if the diffraction at both edges of the propagation band is of opposite signs, then it must vanish to zero at some point within the band, corresponding to some angle of propagation of light beam in [8], or to some transverse velocity of light in the short resonators in [9].

We focus in the present paper on the nondiffractive propagation of light through materials with refractive index periodically modulated in two or three spatial dimensions (in two- or three-dimensional photonic crystals, respectively). The expected phenomenon is illustrated in Fig. 1(c), where the zero diffraction is supposed to occur in a particular point in the wave-vector domain where the curvature of the surfaces of constant frequency becomes exactly zero. Zero diffraction physically means that light beams of arbitrary width can propagate without diffractive broadening and, equivalently, that arbitrary light structures can propagate without diffractive "smearing." The nondiffractive propagation of the light beams in two-dimensional photonic crystals has been shown numerically [10] and experimentally [11] up to now. No analytical studies have been reported. We concentrate in the paper on the analytical study of the phenomenon, allowing to perform an asymptotical treatment, and to get some insights into the physics of wave propagation through periodic media.


FIG. 1. Geometrical interpretation of diffraction of electromagnetic radiation propagating along the $z$ axis: (a) positive, or normal diffraction in propagation through homogeneous materials; (b) negative, or anomalous diffraction; and (c) zero diffraction. The area of negligible diffraction is indicated.


FIG. 2. Harmonic modulation of refraction index as, e.g., imposed holographically in photorefractive material. In this twodimensional case the grating is written by two pairs of counterpropagating beams.

Nondiffractive light propagation can be expected in differently fabricated photonic crystals with different symmetries of the lattices, and with different shapes of inclusions of enhanced (or reduced) refraction index. In this paper we restrict to a simplest case of a harmonic photonic crystal-a material with sinusoidal modulation of refraction index in two- and three- dimensional space. Such harmonic photonic crystal could be produced, e.g., holographically, i.e., by light interference on a photorefractive material [12]. Interference of two pairs of laser beams could result in two-dimensional dynamical photonic crystals as illustrated in Fig. 2. In the present paper we concentrate on a two-dimensional case, however, simulations show that the basic phenomena persist in the case of three-dimensions.

## II. MODEL

We consider a superposition of two periodic lamellaelike refraction index gratings: $\Delta n(\mathbf{r})=2 m\left[\cos \left(\mathbf{q}_{1} \mathbf{r}\right)+\cos \left(\mathbf{q}_{2} \mathbf{r}\right)\right]$ with $\left|\mathbf{q}_{1}\right|=\left|\mathbf{q}_{2}\right|=q$ at angles $\pm \alpha$ to the optical axis, as shown in Fig. 2. This results in refractive index modulation profile $\Delta n(x, z)=4 m \cos \left(q_{\perp} x\right) \cos \left(q_{\|} z\right)$, with $q_{\|}=q \cos (\alpha)$, and $q_{\perp}$ $=q \sin (\alpha)$. The crystallographic axes of such a harmonic photonic crystal are $\pi\left( \pm 1 / q_{\perp}, 1 / q_{\|}\right)$, and the reciprocal lattice vectors of the photonic crystal are $\mathbf{q}_{1}$ and $\mathbf{q}_{2}$. We further assume for simplicity that the spatial period of the photonic crystal is significantly larger than the wavelength of the probe beam. This legitimatizes paraxial approximation for the description of propagation of the probe beams:

$$
\begin{equation*}
\left[2 i k_{0} \partial / \partial z+\partial^{2} / \partial x^{2}+2 \Delta n(x, z) k_{0}^{2}\right] A(x, z)=0 \tag{1}
\end{equation*}
$$

Here $A(x, z)$ is the slowly varying complex envelope of the electromagnetic field in two-dimensional space $E(x, z, t)$ $=A(x, z) e^{i k_{0} z-i \omega_{0} t}$ propagating along the $z$ direction with a wave number $k_{0}=\omega_{0} / c$.

One advantage of the use of paraxial approximation is a substantial simplification of the problem to the explicit analytical expressions in asymptotical cases. Another advantage of Eq. (1) is the possible generalization of the present study to the dynamics of Bose-Einstein condensates in periodic (dynamical) potentials [13]. The disadvantage is that the treatment is restricted for the photonic crystal with the modulation period larger than the wavelength of the probe beam.

First we perform an analytical study of the propagation by expanding the electromagnetic field into a set of spatially


FIG. 3. Transverse dispersion relation obtained by numerical integration of Eq. (3) considering the five most relevant modes ( $j, l=-1,0,1$ ): (a) in absence of modulation $m=0$ (dashed parabolas), and for a weak modulation of refraction index $m=0.003$ (solid line); and (b) for particular amplitude of modulation $m=0.0175$ inducing zero diffraction. Insets show corresponding Bloch modes calculated at $k_{\perp}=0$. Parameters: $\lambda=500 \mathrm{~nm}\left(k_{0}=4 \pi 10^{6} \mathrm{~m}^{-1}\right), q_{\perp}$ $=2 \pi 10^{6} \mathrm{~m}^{-1}$, and $q_{\|}=0.7510^{6} \mathrm{~m}^{-1}$.
harmonic (Bloch) modes. Then we prove the nondiffractive propagation by a direct numerical integration of wave equation (1).

## III. HARMONIC EXPANSION

The technique of our harmonic expansion is analogous to expansion in a finite set of Bloch modes, as, e.g., described in [14]:

$$
\begin{equation*}
A(x, z)=\sum_{j, l} A_{j, l} e^{i k_{\perp, j} x+i k_{\|, l}}, \tag{2}
\end{equation*}
$$

with $\quad \mathbf{k}_{j, l}=\left(k_{\perp, j}, k_{\|, l}\right)=\left(k_{\perp}+j q_{\perp}, k_{\|}+l q_{\|}\right), \quad j, l=\cdots-2$, $-1,0,1,2, \ldots$, respectively. The expansion (2) results in a coupled system for the amplitudes of harmonics:

$$
\begin{equation*}
\left(-2 k_{0} k_{\|, l}-k_{\perp, j}^{2}\right) A_{j, l}+2 m k_{0}^{2} \sum_{r=j \pm 1, p=l \pm 1} A_{r, p}=0 . \tag{3}
\end{equation*}
$$

Solvability of Eq. (3) results in a transverse dispersion relation (the dependence of the longitudinal component $k_{\|}$on the transverse component $k_{\perp}$ of the wave vector). We restricted to the five most relevant harmonics in our study (respectively, nine components for three-dimensional case), consisting of a central component with the wave vector $\mathbf{k}$ $=\left(k_{\perp}, k_{\|}\right)$, and four most relevant modulated (sideband) components with the wave vectors $\left(k_{\perp} \pm q_{\perp}, k_{\|} \pm q_{\|}\right)$, respectively. The transversal dispersion relations as calculated numerically from Eq. (3) are given in Fig. 3. In the absence of refractive index modulation $m=0$ the formal solution of Eq. (3) consists of a set of parabolas [dashed curves in Fig. 3(a)] shifted one with respect to another by the vectors of photonic crystal lattice $\mathbf{q}_{1,2}$. These parabolas represent the transverse dispersion curves for uncoupled harmonic components of the expansion (1). For a nonparaxial description of light propagation these parabolas would be substituted by circles. The modulation of the refractive index $m \neq 0$ lifts the degeneracy at the crossing points and gives rise to band gaps in spatial wave number domain [Fig. 3(a)]. We focus on the appearance of plateaus on the transverse dispersion curves, indicat-


FIG. 4. Zero diffraction curve in space of adimensional parameters $\left(f, Q_{\|}\right)$, as obtained by numerical integration of Eq. (3). Insets correspond to the first branch of diagram ( $K_{\perp}, K_{\|}$) arround $K_{\perp}=0$. The dashed lines indicate the transverse dispersion relation considering the 2 and 3 first terms of the power series expansion.
ing the vanishing of diffraction. For some particular parameters, essentially the geometry of photonic crystal given by vectors $\mathbf{q}_{1,2}$ and modulation depth $m$, the upper dispersion branch, corresponding to the most homogeneous Bloch mode (see insets of Fig. 3), can become nondiffractive, with zero curvature at $k_{\perp}=0$ [Fig. 3(b)]. The insets of Fig. 3 show the field eigenfunctions at a particular propagation distance [the envelopes $A(x, z=0)]$.

Next we use adimensional variables: the transverse wave number of light is normalized to the wave number of transverse modulation of refraction index, $K_{\perp}=k_{\perp} / q_{\perp}$, and the longitudinal wave numbers are normalized by $\left(K_{\|}, Q_{\|}\right)$ $=\left(k_{\|}, q_{\|}\right) 2 k_{0} / q_{\perp}^{2}$. The space coordinates are thus rescaled as $Z=z q_{\perp}^{2} / 2 k_{0}$ and $(X, Y)=(x, y) q_{\perp}$. Two significant parameters remain after the normalization: $f=2 m k_{0}^{2} / q_{\perp}^{2}$ which represents the modulation depth of the plane waves propagating in the photonic crystal, and $Q_{\|}=2 q_{\|} k_{0} / q_{\perp}^{2}$, which is proportional to the angle between the crystallographic axes of the photonic crystal. The diffraction coefficient, i.e., a curvature of the transverse dispersion curve $d_{2}=-1 / 2 \partial^{2} K_{\|} / \partial K_{\perp}^{2}$, becomes zero for a particular relation between parameter values of the photonic crystal. Figure 4 shows the nondiffractive regimes in the two-dimensional parameter space $\left(f, Q_{\|}\right)$, that is obtained on a curve, i.e., on the area of a measure one. The zero diffraction curve is a single valued function $Q_{\|}=Q_{\|}(f)$, as follows from the numerical calculation using the Bloch mode expansion. This means that for a given normalized amplitude of the modulation of refractive index $f$ a particular geometry of the photonic crystal can be always found resulting in nondiffractive propagation. The diffraction is normal (positive) below the zero diffraction curve, i.e., for a relatively small angle between the crystallographic axes of the photonic crys-


FIG. 5. Diffractive (a) and nondiffractive (b) propagation of a Gaussian beam obtained by numerical integration of Eq. (1). The calculation parameters correspond to the point $\left(f, Q_{\|}\right)=(0.28,0.57)$ in Fig. 4. (c) and (d) show initial and final envelopes of the beam (intensity), and (e) shows a magnified area from (b). The real world parameters: $\lambda=500 \mathrm{~nm}, n_{0}=1, m=0.0175, \lambda_{0 \perp}=1.0 \mu \mathrm{~m}$, and $\lambda_{0 \|}$ $=7.0 \mu \mathrm{~m}$. The initial beam waist $W_{0}=2.82 \mu \mathrm{~m}$, with corresponding Rayleigh length: $z_{0}=50 \mu \mathrm{~m}$.
tal, and anomalous (negative) above the zero diffraction curve. The strength of the effect, which is proportional to the size of the plateau, depends strongly on the location along the zero diffraction line, as the insets of Fig. 4 indicate.

The use of more harmonics in the Bloch mode expansion does not alter the upper dispersion branch in the vicinity of $K_{\perp}=0$, at least for the reasonably small amplitude of the refraction index modulation $f \leqslant 1$. We performed the Bloch mode expansion [analogous to Eqs. (2) and (3)] including the "ring" of additional four modes, which has not resulted in the sensible modification of the zero diffraction curve.

An analytical form of the zero diffraction curve is in general complicated. However, a simple asymptotical expression can be found in a limit of weak refractive index modulation $f \ll 1$, where only the homogeneous harmonic, and those with $K_{\|}>0$ [the upper parabolas in Fig. 3(a)] are relevant. The power series expansion of the transverse dispersion relation in this limit results in the analytical expression for the coefficient of diffraction: $d_{2}=1-8 f^{2} /\left(1-\Omega_{\|}\right)^{3}+64 f^{4} /\left(1-\Omega_{\|}\right)^{5}$ $+\cdots$. The asymptotic expressions for the zero diffraction curve (in the limit of $f \ll 1$ ) is thus $\left.Q_{\|}\right|_{d_{2}=0} \approx 1-2 f^{2 / 3}$ $+4 f^{4 / 3} / 3+\cdots$, which matches well with the numerically cal-


FIG. 6. (a) Initial pattern constructed as a composition of the Gaussian beams, and (b) diffracted pattern after a propagation length of $160 \mu \mathrm{~m}$ through an homogeneous space, as calculated integrating numerically Eq. (1) in three spatial dimensions. (c) The pattern propagated in photonic crystal over the same propagation length. The parameters as in Fig. 5.


FIG. 7. Propagation of a Gaussian beam for a fourth order diffraction by integration of Eq. (4) for a two-dimensional case, with $d_{4}=1$ : (a) evolution of the fourth power of the half-width for initial half-widths: $\Delta x_{0}^{4}=0.5,1$, and 2. Dashed line is for guiding the eye; (b) spatial profiles of the initial $Z=0$ (dashed) and the final $Z=0.1$ beam. Note the broadening of the central part and the appearance of the background radiation.
culated curve (Fig. 4) in the limit of $f \ll 1$, and reproduces qualitatively well the behavior (in particular the bending of the zero diffraction curve) for the moderate amplitudes of the modulation $f \propto 1$.

## IV. NUMERICAL SIMULATION

We checked the above predicted phenomenon of nondiffractive propagation by direct numerical integration of Eq. (1) for a two-dimensional modulation of refractive index (Fig. 5) and also for the three-dimensional case. The integration shows an evident decrease of the diffraction: whereas in the absence of spatial modulation of the refractive index the narrow prove beam is diffractively broadening [Fig. 5(a)], in the presence of refraction index grating of particular parameters the spreading was strongly suppressed [Fig. 5(b)]. The used set of parameters $\left(f, Q_{\|}\right)=(0.28,0.57)$ lies close to the analytically calculated zero dispersion curve of Fig. 4 and are chosen in order to obtain a diffraction minimum for the Gaussian envelope [15]. The magnified part of the nondiffractive propagation plot [Fig. 5(e)] indicates that the nondiffractively propagating beam is in fact an envelope of the spatially modulated nondiffractive propagation mode. We note that our calculations in the three-dimensional case result in essentially the same nondiffractive behavior as in twodimensional cases shown in Fig. 5.

We note that in the initial stage of propagation through photonic crystal the injected beam splits into two-a nondiffractive envelope of the corresponding nondiffractive Bloch mode, and a rapidly diffracting envelope of the other (diffractive) Bloch modes. The detailed analysis of the projec-
tion into the Bloch modes will be presented elsewhere.
Figure 6 shows the nondiffractive propagation as obtained by numerical simulations of Eq. (1) in the case of three spatial dimensions, with the refractive index modulation profile $\Delta n(x, z)=8 m \cos \left(q_{\perp} x\right) \cos \left(q_{\perp} y\right) \cos \left(q_{\|} z\right)$. The calculations show that the phenomenon of nondiffractive propagation persists in the case of two transverse dimensions. The calculations are evidence of the use of nondiffractive propagation for the efficient transfer of the arbitrary two-dimensional patterns of light.

## V. ASYMPTOTIC BEHAVIOR IN THE ZERO-DIFFRACTION REGIME

As the diffraction (the curvature of the dispersion curve) becomes exactly zero on a point in wave number domain then, strictly speaking, nondiffractive propagation has a sense for infinitely broad beams only. Beams of finite width occupy an extended area in a wave-vector space [indicated area in Fig. 1(c)] where the dephasing remains negligibly small for a finite propagation distance. The minimal transverse size of a light structure $\Delta x$ which can propagate with a negligible diffractive broadening over a distance L is inversely proportional to the width of the area of negligible diffraction in wave number domain $\Delta k_{\perp}: \Delta x \approx 2 / \Delta k_{\perp}$. The distance of nondiffractive propagation $L$ is related with the size of nondiffractive area by $\delta k_{\|} L \propto 1$, where $\delta k_{\|}$indicates the small variation of the longitudinal component of the wave vector within the nondiffractive area. We analyzed this weakly diffractive propagation by series expansion of the transverse dispersion curve around $K_{\perp}=0: K_{\|}\left(K_{\perp}\right)=\sum_{n=0}^{\infty} d_{n} K_{\perp}^{n}, d_{n}=(1 / n!) \partial^{n} K_{\|} / \partial K_{\perp}^{n}$. As the second order derivative is zero and all odd order derivatives vanish because of symmetry, then the fourth order derivative (fourth order diffraction) is the lowest and most relevant one, resulting in the following evolution equation:

$$
\begin{equation*}
\left(i \partial / \partial Z-d_{4} \partial^{4} / \partial X^{4}\right) A_{B}(X, Z)=0 \tag{4}
\end{equation*}
$$

Here $A_{B}(X, Z)$ is the envelope of the nondiffractive Bloch mode $B(X, Z): A(X, Z)=A_{B}(X, Z) B(X, Z)$. The derivation of Eq. (4) is performed using a multiscale expansion technique, e.g., along the lines described in [16], considering the different spatial scales for the Bloch mode (small space scale) and the envelope (large spatial scale). The application of multiscale expansion [16] leads directly to Eq. (4). Alternatively one can consider $A_{B}(X, Z)$ as the envelope of the Wannier


FIG. 8. (a) The evolution of the width of the beam in propagation through photonic crystal as obtained by numerical integration of Eq. (1) for initial waists $W_{0}=2.82 \mu \mathrm{~m}, W_{0}=3.95 \mu \mathrm{~m}$, and $W_{0}=5.08 \mu \mathrm{~m}$. The same parameters of Fig. 5. (b) Spatial intensity profiles for $W_{0}=2.82 \mu \mathrm{~m}$, the initial Gaussian beam and the final beam after a spatial low pass filter.
functions. Then, as shown in [17], the evolution equation for the envelope of the Wannier function can be written straightforwardly (and strictly mathematically) by a direct substitution of spatial dispersion relation by the operators of spatial derivatives.
$d_{4}$ is an adimensional coefficient for the fourth order diffraction of order of magnitude of one. The asymptotic expressions for the fourth order diffraction (in the limit of $f$ $\ll 1$ ) are $d_{4} \approx 32 f^{2} /\left(1-Q_{\|}\right)^{5}$, and along the zero diffraction curve $\left.d_{4}\right|_{d_{2}=0} \approx f^{-4 / 3}$.

The envelope of the Bloch mode propagates obeying Eq. (4), which as written in spatial Fourier domain reads

$$
\begin{equation*}
A_{B}\left(K_{\perp}, Z\right)=A_{B}\left(K_{\perp}, Z=0\right) \exp \left(i d_{4} K_{\perp}^{4} Z\right) \tag{5}
\end{equation*}
$$

Considering a $\delta$ beam, with a flat spatial spectrum, Eq. (5) yields that the spatial components with $\left|K_{\perp}\right|^{4}>\left|\Delta K_{\perp}\right|^{4}$ $\approx \pi /\left(d_{4} Z\right)$ dephase significantly $(\Delta \varphi \geqslant \pi)$ over a propagation distance Z . Assuming that the coherent (not dephasing) spatial spectrum components $\left|K_{\perp}\right|<\left|\Delta K_{\perp}\right|$ result in a nearly Gaussian beam of half-width $\Delta X \approx 2 / \Delta K_{\perp}$ the following expression for the evolution of the half-width of the beam is obtained: $\Delta X^{4}(Z) \approx 16 d_{4} Z / \pi$, which in terms of the real world parameters reads

$$
\begin{equation*}
\Delta x^{4}(z) \approx 8 d_{4} z /\left(\pi k_{0} q_{\perp}^{2}\right) \tag{6}
\end{equation*}
$$

The evolution of nonzero width Gaussian beams can be directly calculated by convolution of the Gaussian envelope with the envelope of the diffracted $\delta$ beam given by Eq. (6). Figure 7(a) shows the evolution of the half-width of the Gaussian beams of different initial half-width $\Delta x_{0}$ as calculated by solving numerically the propagation equation (4). For large distances of propagation $z>\pi k_{0} q_{\perp}^{2} \Delta x_{0}^{4} /\left(8 d_{4}\right)$ the beam broadening follows the asymptotic fourth root dependence as evaluated above Eq. (6). Figure 7(b) shows the spatial envelope of the subdiffractively propagating beam. The central (weakly broadening) peak consists of in-phase $\left|K_{\perp}\right|<\left|\Delta K_{\perp}\right|$ spatial Fourier components, and the background radiation consists of a dephased spatial Fourier component with $\left|K_{\perp}\right|>\left|\Delta K_{\perp}\right|$.

The asymptotic fourth root dependence has been proved numerically by integrating the initial wave propagation equation (1). Figure 8(a) shows the evolution of the width of the beams with the different initial width of the waists $W_{0}$, and clearly indicates the fourth order power law, as predicted above. The initial increment (and subsequent decrement) of the width is due to the energy projection into nondiffractive
and diffractive Bloch modes, and to the subsequent spreading of the diffractive modes [as seen from Fig. 5(b)]. For the large propagation distances the oscillations of the calculated width of the beam develop, as the diffractive mode, due to the periodic boundaries, fill the whole interaction region, and interfere with the nondiffractive beam. We note that the evolution of the fourth power of the width is plotted in Fig. 8(a), thus the oscillations appear enhanced.

The fourth diffraction coefficient as calculated from the asymptotic slope in Fig. 8(b) is $\left|d_{4}\right| \approx 150$, and corresponds reasonably with that following from the asymptotical analysis $d_{4}=32 f^{2} /\left(1-Q_{\|}\right)^{5}=170$, as calculated for the given parameters. Figure $8(\mathrm{~b})$ shows the typical form of subdiffractive beams consisting of the central coherent peak and the broadened background in a good correspondence with the analytical predictions [Fig. 7(b)].

## VI. CONCLUSIONS

Concluding, we predict the nondiffractive broadening of electromagnetic radiation in propagation through photonic crystals. We calculate the parameters of photonic crystal, and we evaluate the limits of nondiffractive propagation. The limits of nondiffractive propagation depend on the higher order-predominantly fourth order-diffraction, i.e., depend on the position on the zero diffraction curve in parameter space. Assuming for a rough estimation that the spatial scale of refraction index modulation can be in principle reduced down to a half-wavelength of the propagating light $\left|q_{\perp}\right|$ $\approx 2\left|k_{0}\right|$ (which is typical value for photonic crystals), and that the adimensional fourth order diffraction is of order of one, one obtains $\Delta x^{4}(z) \approx z \lambda^{3} /\left(4 \pi^{4}\right)$. This gives a realistic estimation of the minimal width of the nondiffractive beam to be propagated over distance $z$ (or, equivalently, the spatial resolution of the pattern to be communicated). As a numerical example, the nondiffractive propagation length of the beam of half-width of $10 \mu \mathrm{~m}$ and $\lambda=1 \mu \mathrm{~m}$ (the Rayleigh length is about 0.3 mm ) should be of the order of 1 m .

The nondiffractive propagation of light as proposed here can be also realized for Bose-Einstein condensates resting on the countermoving periodic potential lattices.

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